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Source: *Teaching Children Mathematics*, Vol. 23, No. 3 (October 2016), pp. 146-157

Published by: National Council of Teachers of Mathematics

Stable URL: <http://www.jstor.org/stable/10.5951/teacchilmath.23.3.0146>

Accessed: 31-05-2017 19:08 UTC

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Targeting Instruction with Formative Assessment Probes

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In today's classrooms, teachers are called on to gather and use evidence in a timely, formative way to implement instruction that improves mathematics learning:

Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning. (NCTM 2014, p. 53)

Although it sounds straightforward, we know this is far from an easy task. A 2015 Gates Foundation report found that teachers believe in the value and potential of data but face challenges in using it. For example, formative assessment data may highlight broad areas of the Common Core State Standards for Mathematics (CCSSM) (CCSSI 2010) without identifying specific concepts or skills where misunderstandings exist. Moreover, teachers often have difficulty determining how to use the findings to plan instruction or interventions that directly address students' misconceptions (Holmes et al. 2013).

The research

This article introduces the formative assessment probe—a powerful tool for collecting focused, actionable information about student thinking and potential misconceptions—along with a process for



Start with
a strategic
process to
gather and
interpret
evidence of
students'
mathematical
understandings
and
misconceptions;
then aim
your teaching
to address
identified
needs.

targeting instruction in response to probe results. Drawing on research about common student mathematical misconceptions as well as the former work of author Cheryl Rose Tobey (2011), we developed a series of probes and supporting resources to help teachers assess students' understanding of key rational number concepts from CCSSM. These probes are a central feature of three professional development (PD) courses that we created and tested as part of five-year project funded by the National Science Foundation (NSF). The project focused on helping teachers build math content knowledge, diagnostic approaches, and instructional practices for teaching rational number topics.

In this article, we will first help readers get to know more about what a probe is by exploring an example in depth. Then we will use information gathered from a probe to illustrate a process for planning and implementing targeted

instruction for a class of students. To support teachers in using this approach, the article provides examples of probes, student work, and classroom activities as well as a link to a set of probes with accompanying resources.

The tool

A probe is a short, highly focused, quick-to-administer diagnostic assessment designed to pinpoint key foundational ideas that students understand as well as any specific misunderstandings they may have regarding a particular mathematics concept (see **fig. 1**). A probe consists of three to six items designed for students to complete in about 10–15 minutes. Each item requires a two-part response from the student: a selected response and a written explanation using words or pictures. Together, these two pieces of data supply important evidence of a student's understanding and thinking about

FIGURE 1

Each fraction pair below assesses for common misconceptions related to comparing fractions, which helps teachers identify specific understandings and difficulties so they can plan differentiated learning opportunities.

Problems designed to elicit misconceptions and difficulties identified in research	Selected response element: check one $\frac{1}{7}$ ☐ Greater than ($>$) $\frac{1}{10}$ ☐ Less than ($<$) ☐ Equivalent to ($=$)	Explain your choice using words or pictures.	Explanation prompt
	$\frac{6}{8}$ ☐ Greater than ($>$) $\frac{3}{4}$ ☐ Less than ($<$) ☐ Equivalent to ($=$)	Explain your choice using words or pictures.	
	$\frac{4}{5}$ ☐ Greater than ($>$) $\frac{6}{7}$ ☐ Less than ($<$) ☐ Equivalent to ($=$)	Explain your choice using words or pictures.	
	$\frac{5}{12}$ ☐ Greater than ($>$) $\frac{3}{4}$ ☐ Less than ($<$) ☐ Equivalent to ($=$)	Explain your choice using words or pictures.	

The fraction pairs in this probe were carefully selected to assess for common misconceptions related to comparing fractions, such as whole-number reasoning, smaller is always bigger, and gap reasoning.

Sample successful approaches

1. ☒ Greater than ($>$) ☐ Less than ($<$) ☐ Equivalent ($=$)

$\frac{1}{7}$ $\frac{1}{10}$

I think $\frac{1}{7}$ is greater because 7ths are cut in to bigger pieces than 10ths.

This student compares these fractions by thinking about the size of sevenths compared to tenths.

2. ☐ Greater than ($>$) ☒ Less than ($<$) ☐ Equivalent ($=$)

$\frac{4}{5}$ $\frac{6}{7}$

$\frac{6}{7}$ is closer to 1

They are both one away from a whole but $\frac{1}{5}$ away is smaller than $\frac{1}{7}$ away so it's closer to a whole.

This student notices that each fraction is 1 away from a whole and then reasons to determine which is closer to 1 and therefore bigger. In this approach, he is thinking about the size of the "missing" lengths $\frac{1}{5}$ and $\frac{1}{7}$ to conclude that $\frac{6}{7}$ is greater because $\frac{1}{7}$ is a shorter distance away from 1 whole: " $\frac{6}{7}$ is closer to 1. They are both one away from a whole but $\frac{1}{7}$ away is smaller than $\frac{1}{5}$ away, so it's closer to a whole."

Misconceptions to look for

"Whole-number thinking"

Students with this misconception apply whole-number reasoning and make their decision by comparing the digits that make up the fractions. Along with all the other items on the probe, this item elicits evidence of whole-number thinking.

4. ☒ Greater than ($>$) ☐ Less than ($<$) ☐ Equivalent ($=$)

$\frac{5}{12}$ $\frac{3}{4}$

12 is much bigger than 4

"Smaller is always bigger"

Students with this misconception overgeneralize the concept that "the larger the value of the denominator, the smaller the piece" to all cases without considering the numerator (the number of pieces).

2. ☒ Greater than ($>$) ☐ Less than ($<$) ☐ Equivalent ($=$)

$\frac{4}{5}$ $\frac{6}{7}$

The smaller the bottom number the bigger the pieces.

"Gap reasoning"

Students with this misconception either compare the difference between the numerators and denominators or compare the number of pieces needed to make a whole without considering the size of the pieces.

1. ☐ Greater than ($>$) ☐ Less than ($<$) ☒ Equivalent ($=$)

$\frac{4}{5}$ $\frac{6}{7}$

There is one left on each fraction.

"There is one space left on each fraction."

a mathematics topic to inform instruction. This combination helps reveal underlying patterns in incorrect answers and show whether correct selected responses are supported by strong or faulty reasoning.

Each probe has a Resource for Analysis hand-out to guide teachers' interpretation of their

students' work by providing information and examples of successful approaches and misconceptions. The development of the probes and resources was informed by reviewing the research on student learning and misconceptions and by collecting and analyzing thousands of student work samples from two projects funded by

This article showcases the probe in **figure 4** as a tool to gather evidence focused on the number line representation for fractions.

Probes as a tool for targeting instruction: A five-step process

Step 1: Establish a math content focus.

What conceptual math understanding do you want to assess? Why?
What is difficult about this concept for students?

Step 2: Develop a plan to collect evidence.

What probe will you use to gather evidence of student thinking about the ideas related to your content focus?
When will you administer the probe? How?

Step 3: Process the results.

How will you analyze and interpret the student work that you collect?

Step 4: Plan and implement targeted instruction.

What instructional activities are good choices for building on student strengths to address areas of difficulty?
What are ways to differentiate the instruction for the range of learners?

Step 5: Reflect and adjust.

How will you determine what impact the instruction had on student learning?
What next steps will you take based on students' current understanding?

the Institute for Education Sciences (Eliciting Mathematics Misconceptions: <http://em2.edc.org>) and NSF (Building Mathematics Knowledge for Teaching Struggling Learners: <http://edc.org/acessmath>). Although probes are applicable across mathematics content areas and grade levels, this article focuses on fraction examples for grades 3–5.

According to CCSSM, students must be able to compare fractions using different methods and to justify their conclusions (CCSSI 2010, 3.NF.3 and 4.NF.2). The probe shown in **figure 1** assesses students' understanding of comparing fractions that are represented numerically and the reasoning they use to do this. Each pair of fractions in this probe was carefully selected to assess for common misconceptions related to comparing fractions, including *whole-number reasoning*, *smaller is always bigger*, and *gap reasoning* (see **fig. 2**).

The probe's focused design helps teachers identify specific understandings and difficulties so they can plan differentiated learning opportunities that address the needs in the range of learners in a class. For example, students who demonstrate whole-number thinking tend to view the numerator and denominator as separate numbers and would benefit from building foundational understanding of the part-to-whole relationship by using fraction circle manipulatives and other visual representations. Meanwhile, a different instructional focus makes sense for students who successfully compare all the fraction pairs except for $\frac{4}{5}$ and $\frac{6}{7}$, for which they show evidence of gap reasoning (i.e., viewing $\frac{4}{5}$ and $\frac{6}{7}$ as equivalent because each is “one” away from a whole, without considering that the gaps are actually two different-size fractions, $\frac{1}{5}$ and $\frac{1}{7}$). Opportunities to compare similar pairs of fractions by using different approaches, including real-world contexts, would be helpful for these students.

The process

Using probes and other assessments to gather evidence of student thinking is only one piece of the puzzle. Teachers also need a process for acting on the evidence in a strategic way. We suggest a five-step approach that begins with planning for data collection and continues through classroom instruction (see **fig. 3**). We will illustrate this process by using a probe that focuses on the number line representation for fractions. Our article showcases the probe as the tool for gathering evidence, but this process can also be applied to other types of assessments.

An example

Flexible use of the number line representation is recommended and expected in the fraction standards for elementary grades, and such use extends to other rational number content in the middle grades (CCSSM 2010). The Institute of Education Sciences, in its review and synthesis of research on effective fractions instruction, outlines five recommendations for helping students build understanding of fractions, including using number lines as a central representational tool for teaching fractions concepts (Siegler et al. 2010). To support students in using the number line representation for fractions, a teacher may want to know what foundational

understandings students possess and whether they hold any common misconceptions. The teacher could implement the five-step process as exemplified next.

Step 1: Establish a math content focus.

We designed a probe to assess for foundational understandings and common misconceptions related to locating and interpreting fractions on the number line (see **fig. 4**). This probe gathers information about the extent to which students can (1) locate fractions on a number line, (2) understand that a fraction is a number with a magnitude that can be represented by a point on the number line, and (3) interpret intervals and recognize the need for equal intervals when determining the value of a fraction. Students who think flexibly about fractions and the number line can employ successful counting and benchmarking strategies to identify the correct location of specific fractions on the number lines (see **fig. 5**).

Although the probe looks like a familiar number line assessment, it has a distinguishing design feature: We have carefully selected and field-tested the locations of the points to elicit common misconceptions. In particular, this probe targets two areas of difficulty reflected in the research, which we call *use of an incorrect counting strategy* and *use of incorrect benchmarking* (see **fig. 6**).

FIGURE 4

This probe reveals foundational understandings and common misconceptions related to locating and interpreting fractions on the number line.

1. Choose the letter that shows the location of $\frac{3}{8}$.	☐ A ☐ B ☐ C ☐ D ☐ E	
Explain why you chose this location:		
2. Choose the letter that shows the location of $\frac{3}{10}$.	☐ A ☐ B ☐ C ☐ D ☐ E	
Explain why you chose this location:		
3. Choose the letter that shows the location of $\frac{3}{4}$.	☐ A ☐ B ☐ C ☐ D ☐ E	
Explain why you chose this location:		

FIGURE 5

The student work below shows two different successful approaches to the first sample item (from **fig. 4**).

Successful approach 1

Choose the letter that shows the location of $\frac{3}{8}$.	☐ A ☐ B ☐ C ☐ D ☐ E	
Explain why you chose this location.		
I split the line into eighths, and then just counted three.		

This student further partitioned the number line to create equal intervals. Next, the student counted up three intervals from zero. This student shows an understanding that a counting strategy can be in use when the unit has been partitioned to match the size indicated by the denominator. Knowing to create smaller equal intervals shows understanding of making more precise units.

Successful approach 2

Choose the letter that shows the location of $\frac{3}{8}$.	☐ A ☐ B ☐ C ☐ D ☐ E	
Explain why you chose this location.		
Because I know that $\frac{3}{8}$ is less than $\frac{1}{2}$ BUT greater than $\frac{1}{4}$		

This student used a benchmarking approach, comparing the magnitude of $\frac{3}{8}$ to common benchmarks $\frac{1}{2}$ and $\frac{1}{4}$, then using that comparison to determine which point shows the location of $\frac{3}{8}$ on the number line. For this selected-response item, using benchmarks is a good strategy for determining the location of the given fraction.

The probe may look like a familiar assessment, but we carefully selected and field-tested the number line point locations to elicit common misconceptions in two areas of difficulty that the research reflects:

(a) the use of an incorrect counting strategy

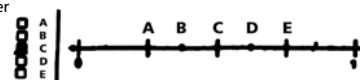
Misconceptions to look for

Incorrect counting

In many surprising ways, students count hash marks or points rather than counting intervals, the space between two hash marks.

Incorrect approach example 1

Choose the letter that shows the location of $\frac{3}{8}$.



Explain why you chose this location.

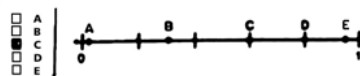
I choose letter C because C is at the third point.

This student may be counting the points: A is 1, B is 2, and C is 3. Or this student might be counting unequal intervals: 0 to 1 is 1, A to B is 2, and B to C is 3.

Students also count without attending to whether the intervals are equal, or they automatically assign the size of the interval on a number line on the basis of the denominator of the given fraction.

Incorrect approach example 2

Choose the letter that shows the location of $\frac{3}{10}$.



Explain why you chose this location.

C is $\frac{3}{10}$ because A is $\frac{1}{10}$ B is $\frac{2}{10}$

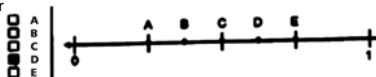
This student counts the points using the given denominators without considering the size of the interval shown.

(b) and the use of incorrect benchmarking.

Incorrect benchmarking

Students sometimes reason incorrectly about the size or location of a fraction relative to common benchmark, such as 0 or $\frac{1}{2}$. They may position the fraction incorrectly in relation to a correctly placed benchmark, incorrectly place the benchmark, or apply incorrect reasoning.

Choose the letter that shows the location of $\frac{3}{8}$.



Explain why you chose this location.

I choose this place because $\frac{1}{2}$ would be C and $\frac{3}{8}$ is after half so I choose D.

This student correctly locates the benchmark fraction $\frac{1}{2}$ but has flawed reasoning about the location of $\frac{3}{8}$ relative to $\frac{1}{2}$.

Step 2: Develop a plan to collect evidence.

After selecting a mathematical focus for gathering evidence and a particular probe, the next step is to administer the probe to students. Typically, probes are given to a class of students in a paper-pencil format. Another option is to use the probes for conducting math interviews with some students, particularly those who struggle to write explanations of their thinking. Tablet and screen recording apps are great

alternative tools for capturing student thinking about probes. The capacity to record encourages students to think aloud about their reasoning and approach. To view a sample student interview and learn more about these tools, visit <http://www2.edc.org/accessmath/resources/diagprobes.asp>

Step 3: Process the results.

Now the teacher is ready to make sense of the

TABLE 1

Analyzing and sorting students' work on the probe gives teachers an overall picture of students' math understandings, difficulties, misconceptions, and class patterns.

Student	Correct—no faulty reasoning	Incorrect counting	Incorrect benchmarking	Incorrect for other reasons	Notes and follow-up What am I wondering about this student's understanding?
Colin		Nos. 2, 3	No. 1		I am wondering about his understanding of the number line. Follow-up: Ask him to locate some whole numbers on a number line and then locate common fractions.
Katrina	Nos. 1, 3	No. 2			Does she realize that it is OK to further partition a number line to get tenths? Follow-up: To help Katrina and other students who had difficulty with no. 2, highlight this partitioning strategy in upcoming lessons.
Tanisha	Nos. 1, 2, 3				Strong explanations. She seems to have a good grasp of locating fractions on number lines with endpoints of 0 and 1. Follow-up: In the next lesson, provide number lines that are more challenging, with varied endpoints, such as $\frac{1}{2}$ and 2.

probe data. Analyzing and sorting students' work on the probe can help the teacher gain an overall picture of individual's math understandings, difficulties, and misconceptions. By sorting work, teachers can identify class patterns and use this information to plan whole-class instruction, create differentiated groupings, and plan individual or small-group interventions. Probe data can be processed by sorting student work into misconception categories—focusing on one item at a time or sorting more holistically across all items. As teachers examine the work samples, they also note questions they would like to ask specific students to find out more about their math thinking (see **table 1**). Teachers can choose to sort and organize the probe results in different ways to fit their instructional and planning preferences and the needs of their students (see **table 2**).

Step 4: Plan and implement targeted instruction.

After analyzing the probe results, when teachers are clear about what conceptual understanding they are working to build they are well situated for the next step in the process: moving from diagnostic to formative assessment by providing

TABLE 2

A class chart with this format is helpful for showing the distribution of students and planning differentiated instruction.

Problem 2 sort			
Correct—No faulty reasoning	Incorrect counting misconception	Incorrect benchmarking misconception	Incorrect for other reasons
Elsa	Marc	Jacob	Tina
James	Katrina	Sheri	D.J.
Caroline	Colin	Gerry	Antoine
Manny	Tami		
Tanisha	Paige		
Peeta	Michaela		
Tim	Justin		
Georgia			
Eli			

targeted learning activities. Assessment is not truly “formative” until the findings are used to inform and implement instruction based on students' needs related to the mathematics

Adapted from the work of mathematics educator Charles Lovitt and the NCTM Illuminations website, this activity has students place fractions on a number line in the form of a clothesline that they can physically fold into equal intervals.

1. The teacher invites two students to hold the clothesline and stretch it across the room. She asks, "What do we have here?"

Some students call out, "A clothesline," and assuming that a number of her students have not seen clothing on a clothesline, she invites more input from students to establish the context. Then she follows up, "Does this remind you of a tool or representation we use in math?"

A number of students answer, "A number line."

Next, she invites students to place the cards 0 and 1, each toward one end of their clothesline.

2. Next students use estimation to place a couple of fraction cards, explaining the reasoning for their selected location. When placing the $\frac{1}{4}$ card, Kieva explains, "I know that one-fourth is half of one-half, so I am placing the fraction card halfway between the zero card and the one-half card."
3. Then the teacher invites students to explain a strategy for more precisely locating the fraction $\frac{1}{4}$.

Justin shares an idea: "We can fold the section of clothesline from the zero card to the one-half card in half and pin the one-fourth card in at the fold." Folding the clothesline helps to build the understanding of equal intervals and the idea of partitioning.

4. After a number of fractions have been placed on the clothesline, the teacher invites students to work in pairs to create a fraction card to add to the clothesline. Students' conversations provide an assessment opportunity for the teacher, and the openness of this part of the task provides some differentiation to meet the needs of the range of learners.

learning goal. By drawing on the probe findings, teachers can plan and implement targeted instruction aimed at presenting students with learning activities tailored to their needs. To give a picture of this step, we share three examples of targeted instruction: two approaches for addressing a counting misconception related to locating fractions on the number line, and one for students who demonstrate strong understanding.

- Use a kinesthetic activity.
Students who show evidence of incorrect counting strategies for locating fractions would benefit from instructional activities aimed at building understanding of the concept of interval on the number line representation. They need to understand why equal intervals are important and be able to interpret the interval; they also need to know that they can further partition a number line to help them locate fractions with denominators that are different from the given intervals. One teacher focused on

building students' understanding in these areas by using the kinesthetic Fraction Clothesline activity (see **fig. 7**).

- Create a problem sequence.
Learning about common student difficulties and misconceptions and their root causes helps teachers chunk the learning into systematic and manageable pieces. An approach that uses key foundational building blocks and combines them in thoughtfully sequenced and scaffolded sets of problems can be an effective intervention for students who demonstrate a misconception. Chunking problems can also help students assess their own understanding or difficulty with the concepts and skills in particular problem types (see **fig. 8**). As part of the process of solving these problems, students are asked to explain their strategies to help solidify their understanding. You could select and sequence problems for students on the basis of their identified needs.

FIGURE 8

Structured to address the incorrect counting misconception, the early problems in this sequence focus on foundational ideas of interval. The sequence builds toward integrating a variety of number line and fraction ideas for naming a fraction on a number line.

Instructional focus

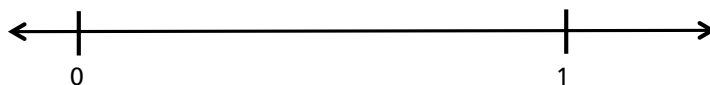
1. Build the important foundational understanding that the interval size is determined by the **number of equal segments or lengths** into which **one unit** has been partitioned.

2. Build understanding of the **importance of the unit** in determining the size of the interval.

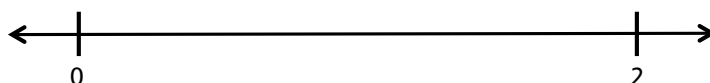
3. Integrate the ideas from the previous problems and apply understanding of part-to-whole fraction ideas to the number line model.

Sample problems

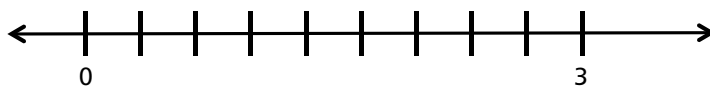
1. Partition the number line into eighths.



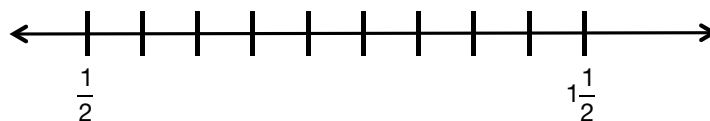
4. Partition the number line into twelfths.



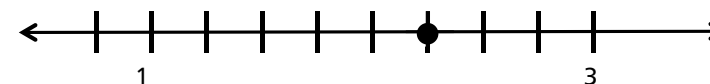
2. Interval: _____



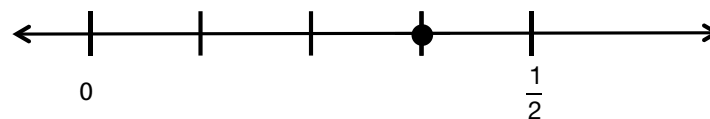
3. Interval: _____



2. Point _____



4. Point _____



- Provide instruction to students who demonstrated strong understanding on the probe. The process of targeting instruction also involves addressing the needs of students who show evidence of strong math understanding on the probe. One approach is to differentiate the activities described above to challenge these students. For the Fraction Clothesline activity, a teacher might have students work in small groups and strategically give some groups challenging sets of fractions to position on their number lines. Another option is to vary the endpoints

of the number lines in the activity and in the problem sequences so they are more challenging than using 0–1. Working with different endpoints will in turn reveal more information about students' understanding beyond what was shown by the probe that has 0–1 number lines only.

Step 5. Reflect and adjust.

To determine how students' thinking has changed as a result of targeted instruction, the teacher and students need opportunities for reflection and assessment. The final step

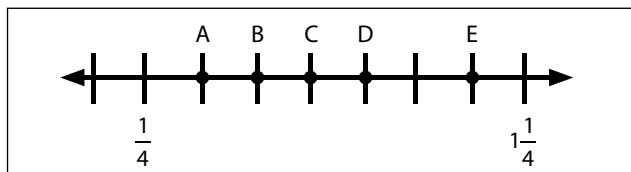
These two examples of probe items increase the difficulty level by varying the endpoints on the number line and using an open-box format.

(a) Probe 6: Number lines with different endpoints

Determine which point shows the location of the fraction.

Choose the letter
that shows the
location of $\frac{1}{2}$.

- ☐ A
☐ B
☐ C
☐ D
☐ E

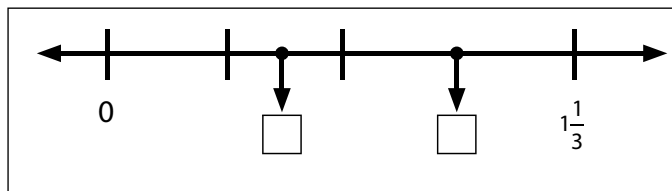


Explain why you chose this location.

(b) Probe 7: Find the fraction or mixed number.

Directions: Fill in the boxes with the fraction or mixed number that best represents the point.

Explain your thinking.



of the probe process is to collect evidence to learn about the impact of the instruction. A few of the ways a teacher could collect evidence of the current status of understanding include the following:

- Administer an alternate version of the original probe.
- Return students' original work on the probe to them and invite them to revise their thinking and reflect on how their thinking has changed.
- Create a short exit ticket with one or two probe items and a reflection prompt.
- Differentiate the post assessment by giving a more challenging probe to students who showed strong understanding on the initial administration (see **fig. 9**).

All four options engage students in reflecting on their learning and provide teachers with valuable information for adjusting their instruction.

Striking the right grain-size and staying focused

In our experience, using formative assessment probes is one way to strengthen teachers' skills in collecting and interpreting evidence of student thinking and leveraging it to target mathematics instruction. The clear and specific focus of a probe yields information of a grain-size that is manageable and actionable. Probe evidence will not tell a teacher everything about a student's understanding of fractions; rather, a particular probe can yield a just-right amount of information about a small slice of conceptual understanding on which a teacher can act strategically to move students' learning forward. We invite you to explore this powerful assessment approach with your students by choosing from a set of probes and accompanying resources provided at <http://www2.edc.org/accessmath/resources/diagprobes.asp>.

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This material is based on work supported by the National Science Foundation (NSF) under Grant

No. DRL-1020163. Any opinion, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of NSF.



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